

1 **Machine Learning Methods for X-ray Scattering Data Analysis**  
2 **from Biomacromolecular Solutions**

3

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8 **Abstract**

9 Small Angle X-ray scattering (SAXS) of biological macromolecules in solutions is  
10 a widely employed method in structural biology. SAXS patterns include  
11 information about the overall shape and low-resolution structure of dissolved  
12 particles. Here we describe how to transform experimental SAXS patterns to  
13 feature vectors and how a simple k-nearest neighbour approach is able to  
14 retrieve information on overall particle shape, maximum diameter ( $D_{max}$ ) as well  
15 as Molecular Mass ( $MM$ ) directly from experimental scattering data. Based on  
16 this transformation we develop a rapid multi-class shape-classification ranging  
17 from compact, extended and flat categories to hollow and random-chain like  
18 objects. This classification may be employed e.g. as a decision block in automated  
19 data analysis pipelines. Further we map protein structures from the protein data  
20 bank (PDB) into the classification space and, in a second step, use this mapping  
21 as a data source to obtain accurate estimates for structural parameters ( $D_{max}$ ,  
22  $MM$ ) of the macromolecule under study based on the experimental scattering  
23 pattern alone, without inverse Fourier transform for  $D_{max}$ . All methods presented  
24 are implemented in a Fortran binary DATCLASS, part of the ATSAS data analysis  
25 suite, available on Linux, Mac, Windows and free for academic use  
26 (<https://www.embl-hamburg.de/biosaxs/software.html>).  
27

## 1 Introduction

2 Small angle X-ray scattering (SAXS) is an increasingly popular method in  
3 structural biology that usefully complements high-resolution structural  
4 techniques such as X-ray crystallography (MX), nuclear magnetic resonance  
5 spectroscopy (NMR) and electron microscopy (EM). SAXS does not require  
6 crystals, labelling or isolated particles at cryogenic temperatures and its  
7 applications extend to the determination of structural parameters, e.g. the  
8 Radius of Gyration ( $R_g$ ) maximum extend ( $D_{max}$ ) and the molecular mass ( $MM$ ),  
9 obtaining the low-resolution shapes of macromolecules and rigid body modelling  
10 of complexes, quantitative characterization of flexibility, as well as time-resolved  
11 conformational changes (1). The scattering intensity  $I(q)$  is recorded as a  
12 function of the scattering vector  $q$ , where the momentum transfer  $q = 4\pi \sin \theta/\lambda$   
13 where  $\theta$  corresponds to half of the angle between incoming and scattered  
14 photons. To determine the scattering of the macromolecule under study, the  
15 background scattering, including sample holder and solvent, typically an  
16 aqueous buffer, has to be subtracted.

17  
18 Over time many methods have been developed to extract relevant information  
19 directly from the experimental scattering intensities, exclusively working with  
20 the experimentally obtained data. In contrast, in this manuscript we consider the  
21 application of Data Mining and Machine Learning (2) to extract structural  
22 information from SAXS data. In short, we shall evaluate the idea that, if there  
23 were a way to locate *similar* macromolecules with known structural parameters,  
24 the parameter values of these similar structures could be used to approximate  
25 the parameter values of the specimen under study. It is to note that in this  
26 context *similarity* shall refer to similarity in scattering patterns, with the  
27 assumption that similar scattering pattern implies similar overall structure, and  
28 not necessarily similar higher resolution detail; the latter may not be the case  
29 (3).

30  
31 For each of the major methods in structural biology curated data banks invite  
32 researchers to deposit models as well as raw data, in particular: the Protein Data  
33 Bank (PDB) (4), the Biological Magnetic Resonance Data Bank (BMRB) (5), the  
34 Electron Microscopy Data Bank (EMDB) (6), and the Small Angle Scattering  
35 Biological Data Bank (SASBDB) (7), respectively. Here, a large number of records  
36 on structural parameters, sequences, shapes, models, and more, have been  
37 accumulated. Using tools like CRY SOL (8) or FoXS (9), theoretical scattering  
38 patterns of atomic models may be readily calculated.

39  
40 Finally we bring the initial idea and available data together by describing  
41 methods on how to make large amounts of data accessible for *Knowledge*  
42 *Discovery*. In particular, in the context of Data Mining and Machine Learning any  
43 measurable property of the specimen under study may be considered a *feature*.  
44 Features describe the input for a machine learning method and may be concrete  
45 values or abstract concepts. In SAXS, the experimental  $R_g$ , the calculated forward  
46 scattering  $I(0)$  as well as the individual experimental intensities at each  $q$  and  
47 any function thereof may be considered potential features. In this manuscript we  
48 shall describe how to represent the overall shape of a protein, e.g. compact, flat,  
49 extended or random chain, with only three shape-related features. Here,

1 random-chains are a mixture of conformations ranging from a compact to fully  
2 extended chains, while extended only refers to preferred extended particles in  
3 solution. Further, to predict structural parameters, a fourth, size-related, feature  
4 may be included in the *feature vector*. The advantage of describing a complex  
5 SAXS pattern in a feature vector of only a few components becomes apparent if  
6 one assumes a form of distance relationship between feature vectors. If two  
7 points in the feature space are close together in the Euclidian sense, then their  
8 properties, i.e. shape and/or structural parameters, should be similar.  
9 Conversely, if they are far apart, their properties should be significantly different.  
10 In order to predict properties of an unknown entity, one may look up its closest  
11 neighbour(s) in the feature space and apply known properties of the neighbour  
12 to the unknown entity. However, the larger the number of components in the  
13 feature vector, i.e. the more dimensions are considered, the more likely are  
14 sparsely populated regions in the underlying data source that could reduce  
15 predictive power, a problem also known as the “Curse of Dimensionality” (10).  
16

17 Here we present a framework of data transformation and feature selection for a  
18 fast and selective lookup of structural neighbours in the space of SAXS patterns.  
19 Based on the proposed feature selection and the source data of the database,  
20 different information may be inferred. In the case of geometrical bodies (11),  
21 simple shapes may be determined quickly, e.g. for use as a proto-shape for *ab*  
22 *initio* modelling, in the case of the PDB (4), structural parameters such as  $D_{max}$   
23 and *MM* of the immediate neighbours, as discussed in this work, but also others  
24 of interest, may be looked up and be used as a starting point for further analysis  
25 and refinement.

## 26 **Methods**

### 27 **Shape classification**

#### 28 **Data Simulation**

29 The command-line program BODIES (11) was modified to simplify the  
30 automated simulation of large amounts of SAXS patterns derived from  
31 geometrical objects with uniform scattering length density of compact spheres,  
32 flat discs, extended rods, compact-hollow cylinders, hollow spheres and flat rings  
33 (Figure 1a). The corresponding dimensions of the geometric bodies, i.e. inner  
34 and outer radius, height, length and width, etc., were uniformly and  
35 independently sampled in ranges from 10 to 500 Ångstrom, respectively.  
36 Classification labels were generated based on the extent of the object, in short:  
37 proportions more or less extreme than 1:4 were considered to define compact,  
38 extended and flat objects, in addition a inner cavity of more than 25% of the  
39 outer radius generally indicates a hollow object. Based on this, 460 000  
40 scattering patterns of various compact, flat, extended, filled and hollow  
41 geometric objects were generated. While clearly limited a selection of body  
42 types, enumerating an exhaustive list of geometrical body shapes would be, at  
43 least, very difficult, especially considering the lack of analytical form factors. As  
44 shown later in the text, classification with *k*-nearest neighbours extends  
45 somewhat outside the boundaries of the mapped class volumes, thus smoothing  
46 out any gaps between geometric objects (Figure 1d). Further, in order to allow  
47 identification of intrinsically-disordered proteins, we employed EOM (12) to

1 generate an additional 560 000 simulations of random chains subsequently  
2 averaged in groups of 20 repetitions to simulate mixtures of flexible proteins.  
3 The lengths of the random chains were selected to follow the size distribution of  
4 amino acid sequences of asymmetric units in the PDB. In total, 488 000  
5 scattering patterns were created across all geometric classes to be used as  
6 training data set for machine-learning classification that encompass basic  
7 geometric objects and disordered polymer chains (Figure 1d).

### 8 **Data Transformation**

9 To normalize for the varying size of objects,  $R_g$  and forward scattering  $I(0)$  were  
10 required. As the generated data is ideal and free of noise, the  $R_g$  was obtained  
11 from the slope of the Guinier plot ( $\ln I(q)$  vs  $q^2$ ) of the first ten computed points  
12 and  $I(0)$  was directly available from the data due to simulation. With these two  
13 parameters, the data was transformed to the dimensionless Kratky scale (13):

$$14 \quad (qR_g)^2 I(qR_g)/I(0) \text{ vs. } qR_g$$

15  
16 Following this, the normalised Porod invariant, or integral  $Q'$ , of the  
17 dimensionless Kratky plot was calculated up to  $qR_g=3$ ,  $qR_g=4$  and  $qR_g=5$ ,  
18 respectively, and expressed as a *normalized apparent volume*, or  $V'$  (14), i.e.

$$19 \quad V' = \frac{2\pi^2}{Q'} \text{ where } Q' = \int_0^{qR_g} (qR_g)^2 I(qR_g)/I(0) dqR_g.$$

20  
21  
22 Each scattering pattern was therefore reduced to three features and its  
23 associated class label (Figure 1b,c). The  $qR_g$  upper bounds were chosen as they  
24 provide a trade-off between contained shape information and the limitations of  
25 the assumption of uniform scattering length density; larger  $qR_g$  values would  
26 separate the point clouds in unrealistic ways (not shown). That said, with the  
27 present selection, the corresponding three-dimensional scatter plot of the  
28 simulated data shows a  $V'$ -space with good separation of the different shape  
29 classes (Figure 1c,d).

### 30 **Learning, prediction, validation**

31 As Figure 1d depicts a well-defined point cloud within the three-dimensional  $V'$ -  
32 space, we added 25 000 randomized points with *unknown* class label to the space  
33 prior to learning. This helped to facilitate compactness of the resulting  
34 predictions, otherwise a query point outside this well-defined  $V'$  would still have,  
35 far away, neighbours and would thus be grouped to a class it does not belong. It  
36 is to note that this random point cloud is not shown in Figure 1d as it would  
37 obscure the actual data of interest.

38  
39 To classify the shape of an unknown entity, its feature vector has to be computed  
40 and the k-nearest-neighbours in the three-dimensional  $V'$ -space are determined  
41 by kd-tree search (15) across the whole training set. Here we chose k=9, partly to  
42 avoid *unknown* classification of the randomly distributed cases, but also to  
43 facility a majority vote classification where classes overlap. The classes of the  
44 neighbours are then weighted by empirical class weights (Supp. Tab. 3) and the  
45 class with the maximum sum of weights is selected as label for the unknown  
46 entity.

47

1 To evaluate the performance of this approach, we used leave-one-out cross-  
2 validation, i.e. we removed each of the 488 000 structures from the source data  
3 in turn and used the remaining data points to predict the class of the removed  
4 one. Cross-validated performance of this multi-class classifier was evaluated by  
5 F1-Measure and Matthew's Correlation Coefficient (MCC) (16).

## 6 Prediction of structural parameters

### 7 Data Generation

8 A snapshot of more than 220 000 asymmetric units and biological assemblies  
9 was taken from the PDB (4). From these we discarded duplicates (i.e. biological  
10 assemblies identical to asymmetric units), entries with nucleotides as well as  
11 peptides with less than 50 amino acids. Entries with more than one MODEL were  
12 discarded unless the models were very similar, in which case we used the first  
13 one listed in the atomic coordinate file. Metals, inorganic molecules and other  
14 post-translational additions were filtered out from all structures. No filtering  
15 was applied with respect to sequence identity, as similarity in sequence does not  
16 always imply similarity in structure (17). From the remaining 165 982 unique  
17 atomic structures, we calculated scattering patterns using CRY SOL (8) using 30  
18 spherical harmonics and 1001 equidistant points up to a  $q_{\max}$  of  $0.6\text{\AA}^{-1}$ . Besides  
19 the calculated scattering pattern CRY SOL also reports a variety of structural  
20 parameters, in particular  $R_g$ ,  $D_{\max}$  and  $MM$ , which we recorded for later use.

### 21 Learning, prediction and validation

22 Similar to the geometric bodies, the  $V$  values were computed for the atomic  
23 structures. Given that for the estimation of structural parameters not only the  
24 shape, but also the size of the molecule is important,  $R_g$  was included as a size  
25 feature in addition to the three  $V$  shape features; here,  $R_g$  was chosen over  $D_{\max}$   
26 as the former can be directly obtained from the experimental data whereas the  
27 latter can usually only indirectly be estimated.

28  
29 To assess the structural parameters of an unknown entity, the feature vector is  
30 computed and the k-nearest structural neighbours, here  $k=5$ , in a four-  
31 dimensional space combining the three dimensions of  $V$  along with  $R_g$ , are  
32 determined by kd-tree search (15). Here, the parameter  $k=5$  was chosen to  
33 minimize the relative prediction error. From this, the parameters, i.e.  $D_{\max}$  and  
34  $MM$ , are estimated as weighted mean of  $D_{\max}$  and  $MM$  of the neighbours, where the  
35 weights correspond to the normalized inverse Euclidean distance to the  
36 unknown entity, i.e. the closer the neighbour, the more important its  
37 contribution to the prediction.

38  
39 To evaluate the performance of this approach, we used leave-one-out cross-  
40 validation, i.e. we removed each of the 165 982 structures from the source data  
41 in turn and used the remaining structures to predict  $D_{\max}$  and  $MM$  of the removed  
42 structure.

### 43 Application of shape classification and prediction of structural parameters to 44 experimental data

45 The classifier was further applied to the 401 public experimental SAXS data sets  
46 without nucleotides, available from SASBDB (7) at the time of writing. As

1 random-chain classifications may potentially indicate modular, flexible or  
2 unfolded proteins, we also collected experimental SAXS data on folded and  
3 chemically-modified unfolded ribonuclease A and folded and denatured Lipase B  
4 at the EMBL P12 SAXS beam line at PETRA-III (18), DESY, Hamburg, Germany, to  
5 compare the results of the random-chain classification with those from  
6 traditional biophysical methods, i.e. CD spectropolarimetry and tryptophan  
7 fluorescence spectroscopy. See Supplementary Material for details on their  
8 preparation.

9 To study the effects of experimental noise on shape classification and prediction  
10 of structural parameters, we further collected experimental data of 100  
11 repetitions of 50 ms exposures of bovine serum albumin (BSA) in 50 mM HEPES,  
12 pH 7.5 buffer. After subtracting 100 buffers from 100 samples, the resulting 100  
13 data sets were identical up to noise as evaluated by CorMap (19).

14 All experimental data was submitted to SASBDB for reference. The following  
15 accession codes were assigned: SASDDK3 (Lipase B), SASDDL3 (folded  
16 ribonuclease A), SASDDM3 (chemically unfolded ribonuclease A) and SASDDN3  
17 (100 repetitions of bovine serum albumin; buffers, samples and subtracted data  
18 were deposited).

## 19 Results

### 20 Shape Classification

21 Appropriate evaluation of multi-class classification systems is itself a topic of  
22 ongoing research. In this work we follow the recommendations of Powers (2011)  
23 and report F1 score and MCC for each shape category (Table 1). Here, F1 is a  
24 measure that considers precision and recall of the classifier with a range  
25 between 0.0 and 1.0, correspondingly MCC determines the correlation between  
26 expected and predicted classes with a range from -1.0 to 1.0. In both cases, larger  
27 (positive) values are associated with better performance. In addition, Supp.  
28 Figure 3 details the confusion matrix, i.e. the actual counts of expected and  
29 predicted classes of the Leave-One-Out cross validation, together with recall and  
30 precision percentages in the margins. Overall accuracy of classification across all  
31 shapes is reported as 96.5%.

	F1 score	MCC
Unknown	0.991	99.1 %
Compact	0.962	95.1 %
Extended	0.969	95.8 %
Flat	0.957	94.7 %
Ring	0.980	97.8 %
Compact-hollow	0.938	93.3 %
Hollow-sphere	0.997	99.7 %
Random-chain	0.964	96.2 %

34 Table 1: F1 score and Matthew's Correlation Coefficient (MCC) for k-nearest  
35 neighbours multi-class classification results of the individual shape categories.

36  
37 Further, we predicted the shape classification of the 165 982 unique atomic  
38 structures of the PDB and visualized the resulting point cloud in  $V^3$ -space (Figure

1 2a). It is immediately apparent that the overall shape of the distribution of  
2 proteins (opaque circles) is very similar to that obtained by geometric objects  
3 (transparent background), with only 25 structures considered outside the  
4 volume mapped by the geometric objects and thus being assigned an “unknown”  
5 class label (open circles). Interestingly, most (~90%) of the PDB structures are  
6 classified as compact/globular, while, for example, more extended proteins are  
7 much less represented (~3%). A different picture arises from experimental data  
8 deposited in SASBDB (Figure 2b). Here the distribution (Supp. Tab. 4) tends  
9 more towards the extended, flat and random-chain area (>50%), reflecting the  
10 fact that solution scattering is often employed for systems that do not easily  
11 crystallize. Indeed, the shape classification of experimental SAXS data may also  
12 be done to describe protein solution state or solution state transitions when the  
13 high-resolution structure is not available or obtainable. For example, Figure 2c,d  
14 show the  $V'$ -space point cloud positions of SAXS data obtained from native  
15 ribonuclease A compared to a final-state completely denatured protein  
16 highlighting the shift from compact to random/flexible shape categories. SAXS  
17 data collected from Lipase B samples that underwent systematic chemical  
18 denaturation shows the ‘denaturation trace’ through  $V'$ -space as the protein  
19 populations unfold at ever-increasing concentration of guanidine hydrochloride.

## 20 Prediction of Structural Parameters

21 Figure 3a,c summarize the results of the Leave-One-Out cross validation for  
22 prediction of structural parameters of the PDB. As the values of the parameters  
23 are derived from the atomic structures, a good agreement may be expected; in  
24 about 90% of the cases the estimate is within 10% of the true value. The  
25 evaluation of experimental data as deposited in SASBDB (Figure 3b,d) is not as  
26 straightforward as the deposited values depend on sample quality, experimental  
27 conditions and the data analysis of the respective researcher. Interestingly,  
28 compared to the results of the PDB, there seems to be a tendency to obtain  
29 somewhat larger  $D_{max}$  values in manual analysis (Figure 3b), which may, for  
30 example, be explained by the influence of the hydration shell.

## 31 Effects of Experimental Noise

32 Figure 4 elucidates the effect of experimental noise on 100 repetitions of bovine  
33 serum albumin; all frames were found similar to each other up to noise as per  
34 CorMap test (19). As depicted in panel (a), the mapped locations of the 100  
35 frames are slightly spread out, but still close together. Histograms of the  
36 estimated structural parameters  $D_{max}$  and  $MM$  are shown in Figure 4(b) and 4(c)  
37 respectively. Again, a spread may be observed, however, the width of the  
38 distributions most likely correlates strongly with the amount of noise present in  
39 the data (not evaluated). Both distributions are centered on values somewhat  
40 larger than what one may expect from strictly monomeric BSA (~100 Ångstrom  
41 and ~67kDa, respectively), but this may be attributed to the presence of a  
42 fraction of dimers in solution (20).

## 43 Discussion

44 Rapid shape classification as presented in this work is a unique approach in the  
45 field of biological SAXS. However, it is obvious that accurate estimates of  $R_g$  and  
46  $I(0)$  are key for appropriate transformation of experimental SAXS data to  $V'$ -  
47 space. Interestingly, misspecification of these parameters will often result in a

1 data point outside the body of shape space as depicted by Figure 1(d) and  
2 consequently lead to an “unknown” classification; therefore the shape  
3 classification may also be used as an initial validation of  $R_g$  and  $I(0)$ . Further, it  
4 has applications as a building block for automated data analysis (21, 22, 23), e.g.  
5 to decide whether *ab initio* shape modeling or ensemble optimization should be  
6 applied. In addition, shape modeling applications may use the initial  
7 classification as a starting point for their models; DAMMIF (24) has already been  
8 modified to not only use a start model based on the classification, but also to  
9 adapt the search and annealing parameters, e.g. by enabling anisometry  
10 penalties for extended or flat objects.

11  
12 Similarly, at present  $D_{max}$  may only be obtained by inverse Fourier transform of  
13 the experimental scattering pattern, which may be difficult to determine  
14 accurately (25, 26). The presented method provides an independent  $D_{max}$   
15 estimate from similar entries in the PDB based on experimental data alone.  
16 Consequently, this approach may be applied to obtain a starting estimate of  $D_{max}$   
17 for the indirect Fourier transform, or, as a tool for quality assessments during  
18 data deposition procedures, e.g., to SASBDB, whereby the automated  $D_{max}$   
19 estimates may be compared to submitted values for validation purposes (Figure  
20 3a).

21  
22 In the past, multiple concentration-independent methods to determine the *MM*  
23 of biological macromolecules from SAXS data have been established (14, 27, 28),  
24 each with their own respective strengths and weaknesses. In this manuscript we  
25 report the results of the size-and-shape based database lookup method (Figure  
26 3b) without attempting to directly compare with any of the established methods.  
27 The interested reader may find a thorough, comprehensive and quantitative  
28 comparison of all four methods elsewhere (29).

29  
30 It is to note that some details of the presented method were empirically  
31 determined, e.g. the  $qR_g$  integration limits for  $V'$ : while the general magnitude is  
32 appropriate, e.g. on the lower end integration to  $qR_g=1$  corresponds to the  
33 Guinier range and on normalized scale the integral is a constant up to rounding  
34 errors. Consequently, on the higher end  $qR_g=10$  would correspond to wider-  
35 angle i.e. higher resolution information that is not easy to rationalize in terms of  
36 overall parameters. Thus the selected  $qR_g$  values of 3, 4 and 5 are reasonable, but  
37 not necessarily optimal. For example, we chose  $N=3$  integration limits also for  
38 the ease of display. A different selection of limits in number and magnitude might  
39 result in an improved predictive performance. Along the same line of argument  
40 one may observe that in many machine-learning applications it is required to  
41 normalize, scale or transform the training data prior to learning and prediction  
42 to achieve a good predictive result. Here, we used the data “as-is”, however, it is  
43 possible that there is a transformation function that minimizes the relative error  
44 and/or (root) mean square error of the prediction. Potential avenues of  
45 investigation for the  $k$  nearest neighbours method include: (a) selection of  $k$  and  
46 the applied distance weights (b) arbitrary linear and non-linear data scaling and  
47 transformation prior to learning; (c) metric selection and metric learning (30);  
48 (d) and, of course, any other learning method as regression functions, Support  
49 Vector Machines, Neural Networks, Deep Learning, etc. As in this manuscript we

1 focus on outlining and introducing a novel approach, we did not exhaustively  
2 investigate all these options; however, the classifier as presented here is already  
3 on par with established methods (29).

#### 4 **Conclusion**

5 In this manuscript we present a conceptually new approach to rapidly analyze  
6 the scattering patterns in biological SAXS, not as an isolated data point, but in the  
7 context of all known biological macromolecules. We have outlined and described  
8 a simple data transformation that combines large amounts of SAXS data into a  
9 few numbers that suggest themselves as coordinates in a feature space for  
10 machine learning. This space simplifies and improves lookup of similar  
11 scattering patterns in a large dataset. The presented approach of integrating the  
12 intensities has a strong advantage over the methods based on actual  
13 (normalized) intensity values. Our method is independent of the spacing of the  
14 available data points, obviating the need for interpolation to a common grid and  
15 fluctuations of individual intensities have less of an effect for lookup due to the  
16 integration, thus also avoiding the “Curse of Dimensionality”.

17  
18 The techniques described here allow for rapid shape classification and provide  
19 estimates of  $MM$  and  $D_{max}$  with good accuracy. It is to note that so far  $D_{max}$  was  
20 only available indirectly through inverse Fourier transform, but with the new  
21 approach it is now also accessible from experimental data directly. Further, the  
22 general approach as described easily extends to additional parameters of  
23 interest extracted from source data as labels may be assigned arbitrarily.

24  
25 The method has been implemented in the program DATCLASS, integral part of  
26 the ATSAS data processing and analysis suite (31) which is freely available for  
27 academic users (<https://www.embl-hamburg.de/biosaxs/software.html>).

#### 28 **Author Contributions**

29 The initial idea was conceived of and all developments were done by D.F.  
30 Experimental data was collected by C.M.J.. D.F, C.M.J and D.I.S. participated in  
31 critical discussion and wrote the manuscript.

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#### 37 **Supporting Citations**

38 References (32-34) appear in the Supporting Material.

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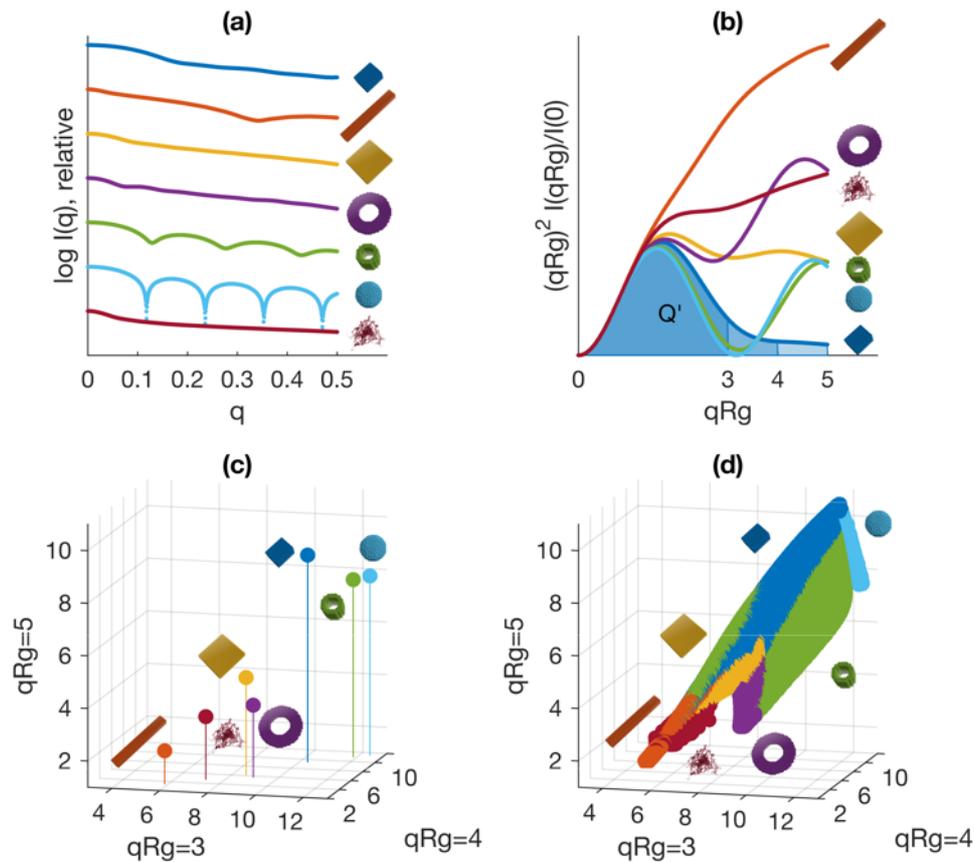
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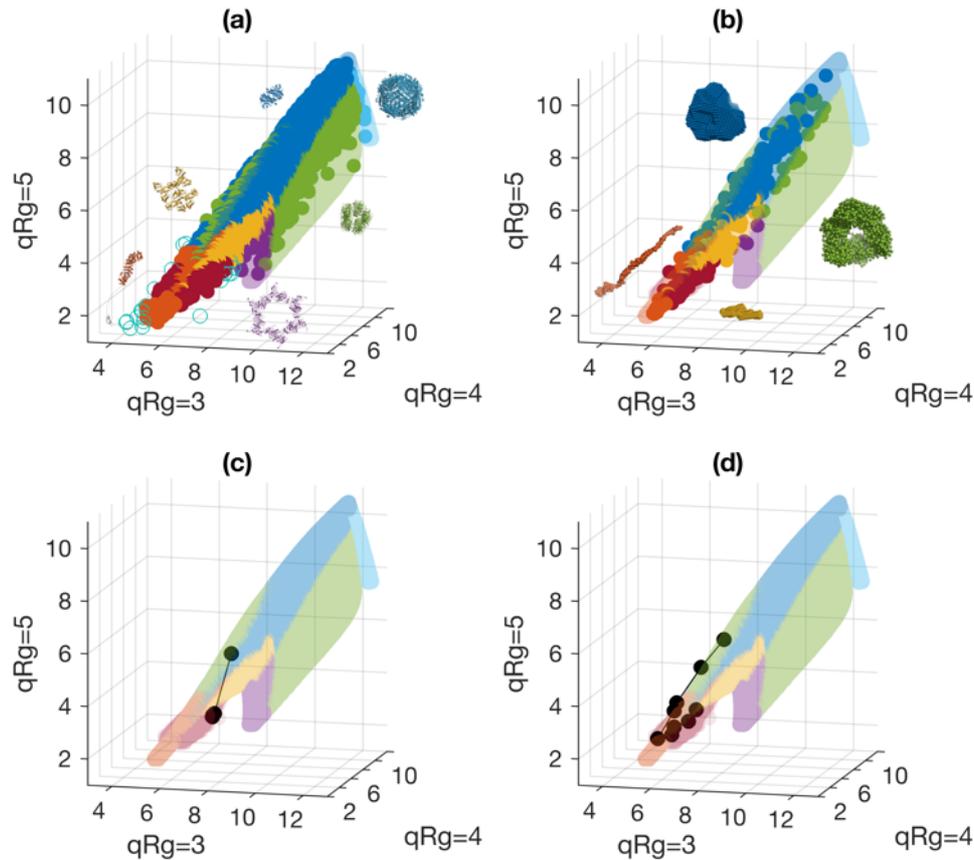
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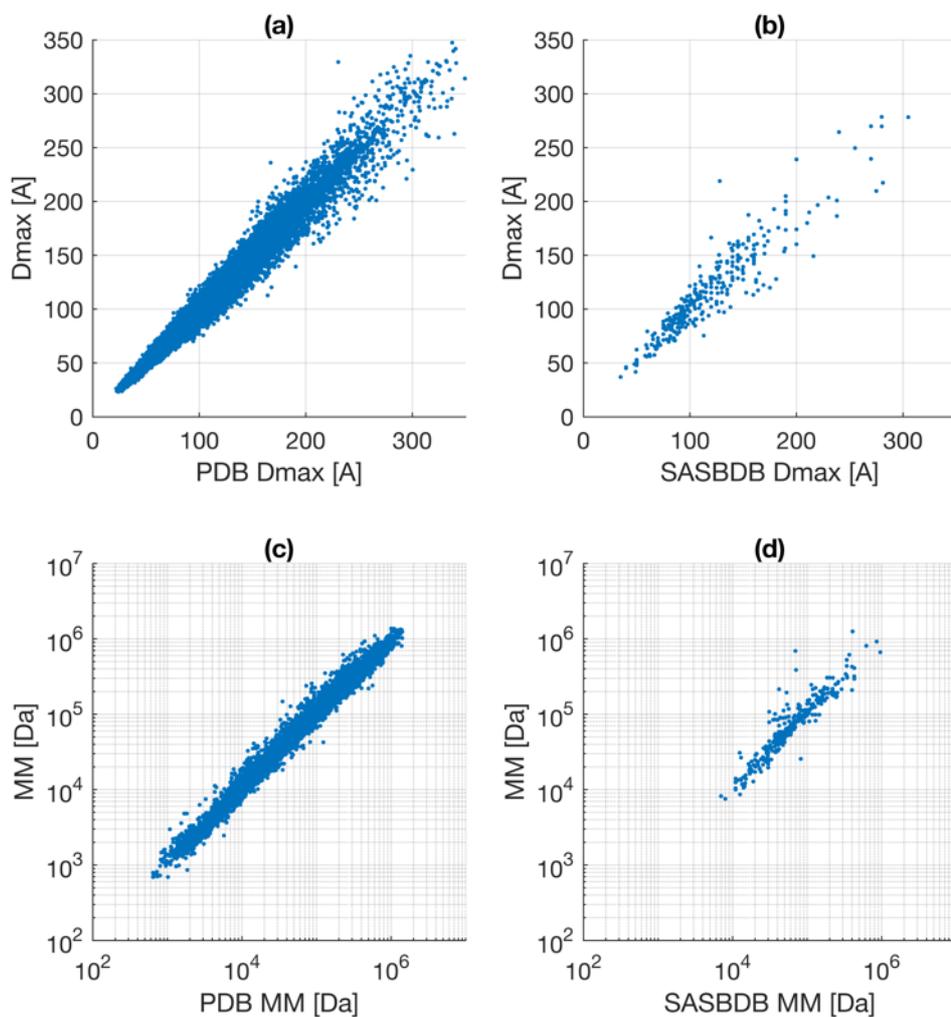
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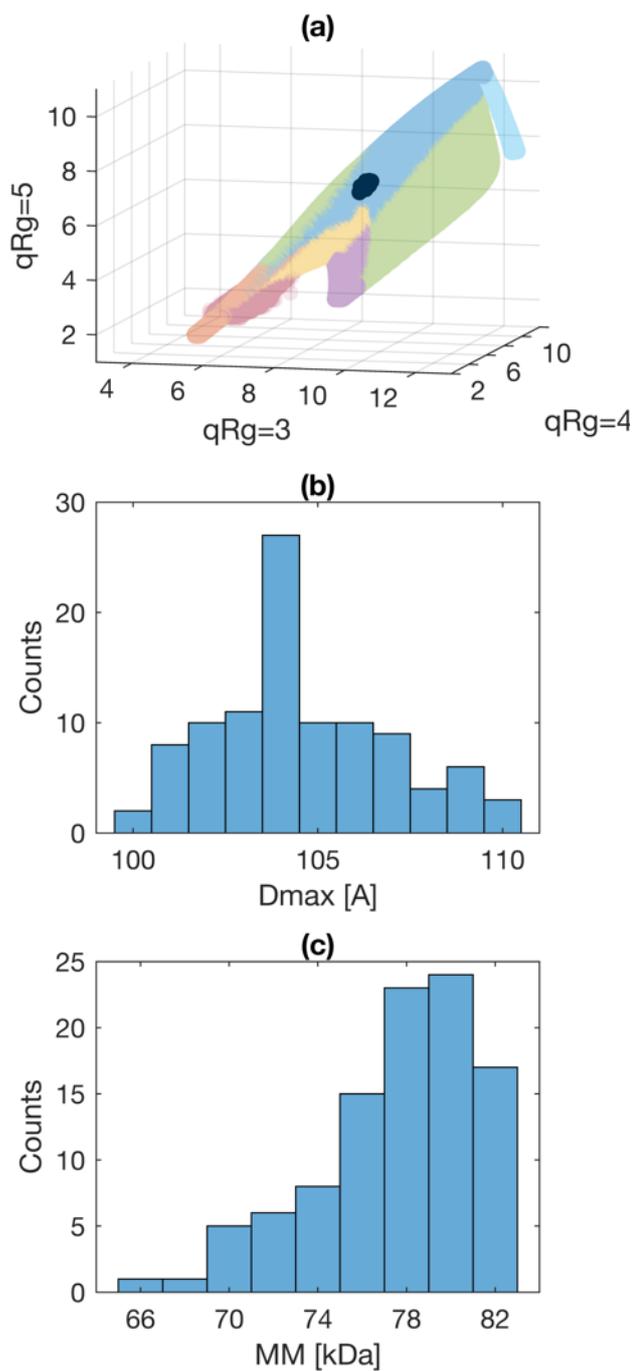
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2 Figure 1: Transformation of scattering patterns of geometric objects and  
3 random-chain on arbitrary log scale (a) via integration of the normalized Kratky  
4 Plot (b) to  $V'$ -space (c, d). Panels (a,b,c) depict a randomly selected member of  
5 each object class, while panel (d) shows the locations of all 488.000 scattering  
6 patterns generated. Color assignments are identical in all panels: compact (dark  
7 blue), extended (orange), flat (yellow), ring (violet), compact-hollow (green),  
8 hollow-sphere (light blue) and random-chain (dark red), also indicated by  
9 corresponding pictograms.  
10



1  
 2 Figure 2: Distribution of (a) atomic structures of the PDB and (b) experimental  
 3 scattering data from SASBDB (opaque) indicating a good agreement of the  $V'$ -  
 4 space mapped out by shapes (transparent) and that covered by atomic  
 5 structures and experimental data. The open circles in (a) depict classifications  
 6 with an 'unknown' class label; structures and models displayed in panels (a) and  
 7 (b) were randomly chosen and placed for the purpose of illustration (PDB: 12as  
 8 (compact), 1v18 (extended), 3oei (flat), 3h3w (ring), 4avt (compact hollow),  
 9 3a68 (hollow sphere), and 2kzw (unknown); SASBDB: SASDA52 (compact),  
 10 SASDA57 (extended), SASDAY4 (flat), SASBDB7 (compact hollow)). Panels (c)  
 11 and (d) show the locations of experimental data of chemically unfolded  
 12 ribonuclease A and lipase B, respectively. The  $V'$ -space trace for ribonucleaseA  
 13 shows the position of the native, folded protein (compact) compared to the  
 14 chemically unfolded final state (random/flexible). The trace for Lipase B shows  
 15 the effect of systematically unfolding the protein population through a  
 16 denaturation gradient of guanidine hydrochloride, from compact to extended  
 17 until a random-chain conformation is reached (see Supplementary Methods for  
 18 details). Color assignments are identical to those of Figure 1.  
 19



1  
2 Figure 3: Estimates of  $D_{max}$  (a, b) and  $MM$  (c, d) for entries of PDB (a, c) and  
3 SASBDB (b, d). In the case of the PDB the expected values are known and a good  
4 agreement can be observed, in about 90% of the cases the estimate is within  
5 10% of the expected value (a,c). No such claim can be made in the case of  
6 SASBDB as the expected values obtained depend on the type of the experiment,  
7 the sample quality, and the data analysis of the submitter.  
8



1  
2 Figure 4: Locations of shape classification in  $V'$ -space (a) and histograms of  
3 structural parameters (b,c) of 100 repetitions of bovine serum albumin which  
4 are identical up to noise. Although affected by the experimental noise, all frames  
5 map closely together in  $V'$  space (a), the estimates of  $D_{max}$  vary from 100 to 110A  
6 (b) and  $MM$  from 66 kDa to 82 kDa (c).  
7